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MANIPULAITVES IN THE SECONDARY MATHEMATICS CLASSROOM USING A TRADITIONAL ALGEBRA TEXT

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A Research Paper Submitted to the Faculty of the
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE IN MATHEMATICS

BEMIDJI STATE UNIVERSITY
Bemidji, Minnesota, USA

August 2010

STATEMENT BY THE AUTHOR

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MANIPULAITVES IN THE SECONDARY MATHEMATICS CLASSROOM USING A TRADITIONAL ALGEBRA TEXT

Christina Mutnansky

This paper is a review of the research available on manipulative use in the mathematics classroom. Mathematical manipulatives, objects that are used to aid a student in learning a mathematical concept, provide a way for students to learn abstract mathematical concepts in a non lecture situation through hands-on learning and discovery. It specifically focuses on the use of manipulatives in the secondary Algebra classroom and how they can be integrated into a traditional curriculum. It discusses the different types of manipulatives that are available as well as reasons why instructors do not use manipulatives. Guidelines are given for their proper use for both students and instructors.

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ACKNOWLEDGMENTS

I would like to thank Dr. Glen Richgels for all of his help throughout this entire process and for keeping me calm whenever I got worked up about something. I would also like to thank Dr. Derek Webb and Dr. Todd Frauenholtz for serving on my committee with a special thank you to Dr. Todd Frauenholtz for all of his wonderful feedback. I would also like to thank everyone else that helped throughout all of my graduate level courses to get me where I am today. Finally, I would like to thank my family. I would like to thank my parents and my husband for all their support and my children for loving their mother and being who they are. They have not made this easy, but they have given me an insight to the learning process first hand that I never otherwise could have experienced.

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Chapter 1: Introduction

The author teaches at Bemidji High School (BHS) which is attended by students from all levels of mathematics. BHS offers mathematics courses from Pre-algebra through AP Calculus (BC). Throughout my teaching career there has been one group of students I have wanted to help succeed, not just get by, more than any other: the students in our beginning algebra course. The students that start their high school mathematics career in this course are not typically high achieving mathematics students. It is normal for them to have had a negative experience in one of their previous mathematics classes, to dislike mathematics, or feel that it is boring.

I wanted to change something in my teaching methods to try to get the students interested. Most of them pass the class, but that is where it stops. They do what they need to do to get by, but oftentimes, there is little retention, and their opinion of mathematics more than likely has not changed at all at the completion of the course.

When I started teaching, I conformed to the methods of the BHS mathematics department: lecture from the book. As time went on and technology changed, I began to teach the students to rely heavily on their calculators. This was not the answer either. Recently I began to try to find a balance: direct instruction, group work, calculator methods—taught from a different approach—and activities with manipulatives. This last addition to my teaching methods is in its infant stages, but I feel I am now headed in the right direction.

Haas (2005) defines manipulatives, models and multiple representations as techniques for generating or manipulating representations of algebraic content or processes, whether concrete, symbolic, or abstract. Manipulatives can be both concrete and virtual. The term concrete manipulative usually implies something physical being made of plastic, wood, foam, etc. that a student can hold and manipulate (Clements, 1999; Curriculum, Technology, & Education Reform, 2008; Eisenberg & DiBiase, 1996; Jones, 1986). Lindroth (2005) defines a virtual manipulative as an online computer manipulative that helps students convert abstract ideas into a particular mathematical model. Using manipulatives provides a way for students to learn mathematical concepts in a non lecture situation through hands-on learning and discovery (Burns, 2004; Curriculum, Technology, & Education Reform, 2008; Lindroth, 2005). Different hand-held manipulatives can be purchased or created by the instructor, while virtual manipulatives can be found throughout the internet on various web sites (Craighead, 2004; Lindroth, 2005).

Statement of the Problem

Dienes (1967) observed that mathematics students were becoming masters in the art of symbolic manipulation—many of them without the slightest idea of what was represented—and thus tried to incorporate the use of manipulatives in the instruction of mathematics. Both concrete and virtual manipulatives seem to be prevalent throughout elementary mathematics today (Eisenberg & DiBiase, 1996; Jones, 1986; Resnick et al., 1998; Stewart, 2003; Waite-Stupiansky & Stupiansky, 1998; Zuckerman, Arida & Resnick, 2005). The use of manipulatives continues, but often begins to wane, toward middle school. Finally, at the secondary level the use of manipulatives may be nonexistent (Stewart, 2003). The use of manipulatives is often considered a time consuming instruction method that is not always fully understood (Curriculum, Technology, & Education Reform, 2008; Lindroth, 2005). As the mathematical ideas become more abstract, the instruction method follows and becomes more abstract as well (Stewart, 2003) utilizing fewer and fewer manipulatives.

Research Questions

I. What are the different types of manipulatives?

II. How should manipulatives be used?

  a. What are the pros and cons surrounding manipulative use?

III. What is the impact the use of manipulatives has on student achievement in the secondary mathematics classroom?

IV. What types of concrete and virtual manipulatives could be used in an introductory algebra classroom with a traditional algebra curriculum?

Significance of the Research Problem

Due to the passing of the No Child Left Behind Act in 2001 (NCLB), the State of Minnesota has had to redefine our mathematics standards. The required State test in mathematics for the class of 2009 was the Basic Skills Test (BST), but the requirements have changed for the class of 2010 and beyond. Schools are now faced with the challenge to help all students succeed with the complexities of algebra (Chappell & Strutchens, 2001; Haas, 2005). In the eleventh grade students will have to pass the Graduation Required Assessment for Diploma (GRAD) test in mathematics in order to graduate and receive a diploma from the state of Minnesota. The GRAD test is written at a much higher level than the BST. The BST measured basic knowledge and skills in mathematics asking students to solve problems derived from common adult situations while the GRAD measures number sense; patterns, functions, and algebra; data, statistics and probability; and spatial sense, geometry, and measurement. A special exception was created for students in the classes of 2010 through 2014 stating that those who do not pass the mathematics GRAD test after attempting it three times may still graduate if they meet all other specified requirements (Larson, 2009). This is a high level, high stakes test, and all students need to be prepared for this exam regardless of their ability level in order to have the greatest opportunity for success in their future.

Limitations and Assumptions

There is a substantial body of research on the effects the use manipulatives have on student achievement. There is an assumption that this effect will apply to all areas and levels of mathematics. Another assumption will be that all of the high school students will be mature enough to use manipulatives in a high school algebra class. Third, there will be an assumption that all students involved are capable of higher level mathematic thinking. Finally, there is the assumption that an instructor will be able to administer the use of manipulatives correctly without formal training.

Manipulatives exist for all areas of mathematics. This study will be limited to the specific manipulatives that apply to the content in the Algebra IA course at Bemidji High School.

Definition of Terms

Adequate Yearly Progress (AYP):

> AYP is a means of measuring the achievement of NCLB goals through standards and assessments. The goal of NCLB is to ensure that all children have a fair, equal, and significant opportunity to obtain a high-quality education and reach, at a minimum, proficiency on challenging state academic achievement standards and state academic assessments. (Minnesota Department of Education, 2010).

Applet

An applet is a web based program that is embedded within a web page. It is written in the Java programming language and is included in the HTML of a web page (Applets, 2010).

Basic Skills Test (BST):

The BSTs are tests of reading, mathematics and writing that students who entered grade 8 in 2004-05 or earlier must pass to receive a diploma from a Minnesota public high school. These reading and mathematics tests were first administered to students when they were eighth graders and the writing test when they were tenth graders. Students entering eighth grade in 2005-06 or later do not take the BSTs (Minnesota Department of Education, 2008).

Concrete Manipulative:

A concrete manipulative refers to a touchable and moveable object which can be reflected on (Curriculum, Technology, & Education Reform, 2008).

Digital Manipulative:

A digital manipulative is a computationally enhanced version of a concrete manipulative (Resnick et al., 1998; Zuckerman, Arida & Resnick, 2005). These manipulatives are also known as tangible (user) interfaces (TUI) (Marshall, 2007).

Direct Instruction

Haas (2005) defines direct instruction as a teaching method which creates a direction and justification for learning by relating new concepts to previous knowledge. Instructors lead students through a precise sequence of instructions based on previously calculated steps that either introduce or reinforce a concept.

Throughout the process instructors provide students with feedback on their progress.

Dynamic Illustrations:

Eremin (2007) defines a dynamic illustration to be unlike static pictures or snapshots. It uses Java , Flash, or some other type of technology to allow students to watch something develop live which often demonstrates things that cannot be shown by conventional methods.

Graduation Required Assessment for Diploma (GRAD) Test:

Embedded in the mathematics Minnesota Comprehensive Assessments – Series II (MCA IIs) are the GRAD mathematics test items. Students' GRAD test scores and MCA II test scores are reported separately. The GRAD test scores determine whether students graduate in the state of Minnesota (Larson, 2009)

Manipulative:

A manipulative is something that a student can hold and manipulate (Curriculum, Technology, & Education Reform, 2008). With the recent advances in technology, this definition has also been expanded to cover the use of digital and virtual manipulatives as well.

Minnesota Comprehensive Assessments Series II (MCA-IIs):

The MCA-IIs are the state tests that help districts measure student progress toward Minnesota's academic standards and meet the requirements of NCLB. The reading and mathematics tests are used to determine whether schools and districts have made AYP. Reading and mathematics tests are given in grades 3-8, 10 and 11. Science tests are given in grades 5 and 8 and once in high school, depending on when students complete their life sciences curriculum (Minnesota Department of Education, 2008).

No Child Left Behind Act (NCLB) of 2001:

NCLB reauthorized the Elementary and Secondary Education Act—the main federal law affecting education from kindergarten through high school. Proposed by President Bush shortly after his inauguration, NCLB was signed into law on January 8th, 2002. NCLB is built on four principles: accountability for results, more choices for parents, greater local control and flexibility, and an emphasis on doing what works based on scientific research (U. S. Department of Education, 2008).

Virtual Manipulative:

Online computer manipulatives (Lindroth, 2005).

Summary Statement

Mathematical manipulatives, objects that are used to aid a student in learning a mathematical concept, provide a way for students to learn abstract mathematical concepts in a non-lecture situation through hands-on learning and discovery (Burns, 2004; Curriculum, Technology, & Education Reform, 2008; Lindroth, 2005).

Both concrete and virtual manipulatives seem to be prevalent throughout all of elementary mathematics, but are uncommon at the secondary level (Eisenberg & DiBiase, 1996; Resnick et al., 1998; Stewart, 2003; Waite-Stupiansky & Stupiansky, 1998). The use of manipulatives is often considered a time consuming instructional method that is not always fully understood (Curriculum, Technology, & Education Reform, 2008; Lindroth, 2005). As the mathematical ideas become more abstract, the instruction method does as well (Stewart, 2003) utilizing fewer and fewer manipulatives. This trend has been changing over the last decade due in part to the technological advances.

Chapter 2: Review of the Literature

Types of Manipulatives

Manipulatives have a dignified presence in American elementary education (Eisenberg & DiBiase, 1996), but the use of manipulatives as part of the learning process is a relatively new idea in the history of education. It was not until the eighteen hundreds that educators began to see the importance and utilize manipulatives in the classroom (Resnick et al., 1998).

Mathematical manipulatives are objects that are used to aid a student in learning a mathematical concept by manipulating it in a visual manner. The use of manipulatives is not a guarantee of success (Eisenberg & DiBiase, 1996), but they help to give students a concrete understanding of abstract mathematical concepts (Balka, 1995; Burns, 2004; Curriculum, Technology, & Education Reform, 2008; Lindroth, 2005). Manipulatives can serve several different purposes: to introduce and develop new concepts, to present a particular problem, and to be used as a problem solving tool (Burns, 2004).

Manipulatives can be both concrete and virtual. The term concrete manipulative usually implies something that a student can hold and manipulate (Curriculum, Technology, & Education Reform, 2008; Eisenberg & DiBiase, 1996). Concrete manipulatives serve two purposes: to model real world things, objects and physical structures and to encourage exploration to try to make abstract manipulations easier (Marshall, 2007; Zuckerman et al., 2005). Until recently, these types of manipulatives were designed and marketed ready for classroom use with little room for student creativity (Eisenberg & DiBiase, 1996).

The development of computerized manipulatives resulted from recent developments in technology, thus allowing more abstract concepts to be taught to students at a younger age. Without the incorporated technology, there are many important concepts that are difficult, or even impossible, to teach using basic concrete manipulatives (Resnick et al., 1998). In the computing world, these manipulatives are referred to as tangible interfaces (Marshall, 2007). These digital manipulatives, concrete manipulatives with embedded computational power, have expanded the range of concepts that were taught through the manipulation of abstract symbols using formal methods and made them available to many more students (Resnick et al., 1998). Today concrete manipulatives can be as creative and complex as LEGOS that you can use to build into a robot then program with a computer (Eisenberg & DiBiase, 1996; Marshall, 2007; Resnick et al., 1998; Zuckerman et al., 2005).

Again with the recent developments in technology, education has seen the development of virtual manipulatives, also known as online computer manipulatives. Similar to concrete manipulatives, virtual manipulatives help students develop a concrete understanding of abstract mathematical symbols (Lindroth, 2005). The Internet has evolved with an abundance of web sites containing various mathematical tools such as simple calculators, dynamic illustrations, interactive environments, and virtual manipulatives that are readily available for classroom use (Craighead, 2004; Lindroth, 2005). Often virtual manipulatives are not as constrained as concrete manipulatives and allow room for more creative participation (Eisenberg & DiBiase, 1996) as well as lend themselves to more extensions (Clements; 1999). Visual manipulatives are said to have an advantage over their technological counterpart digital manipulatives in terms of storage space and set up time (Marshall, 2007). Clements (1999) points out several other advantages: some of the virtual manipulatives allow students to save their place so that they can pick up where they left off as opposed to starting over again each class period: others will give you feedback or replay the process allowing for further exploration; and a very distinct advantage is the ability to link active experience with objects to symbolic representations. When using virtual manipulatives, you must be prepared as you are relying on properly working computers, internet connection, and the site you are accessing (Deubel, 2010).

Use of Manipulatives

The use of manipulatives is often considered a time consuming instruction method that is not always fully understood (Chappell & Strutchens, 2001; Curriculum, Technology, & Education Reform, 2008; Lindroth, 2005; Stewart, 2003). Manipulatives have been perceived as a foreign language with no connections being made to previously learned material (Balka, 2005; Chappell & Strutchens, 2001). It is a concern that students will become reliant on the manipulatives and will not be able to make the bridge to the corresponding abstract concepts (Clements, 1999; Duebel, 2010; Raymond & Leinenbach, 2000). Manipulatives have been avoided due to various reasons: lack of funds to purchase manipulatives (Stewart, 2003); not having adequate numbers of manipulatives to meet classroom needs; considerable amount of time it takes to get them ready for the students and then putting them away again; the difficulties of having them available for at home use (Lindroth, 2005); the need to spend excessive amounts of time on only one or two problems; the difficultly for the students to learn to use as well as the instructors; and finally, manipulatives are often not used in the higher levels as it is believed by many that they are “only for young children” (Curriculum, Technology, & Education Reform, 2008).

The use of manipulatives is very prevalent at the elementary level (Eisenberg & DiBiase, 1996; Resnick et al., 1998; Stewart, 2003; Waite-Stupiansky & Stupiansky, 1998; Zuckerman, Arida & Resnick, 2005), but the affect that the use of manipulatives has on students differs among the grade levels due to the belief that manipulatives are viewed as child’s play (Chappell & Strutchens, 2001). Burns (2004) wrote that it is important manipulatives are not limited to the early grades but are made equally available to older students. After their research on manipulatives in the classroom, Eisenberg & DiBiase (1996) made some interesting observations. They reported students below age twelve enjoyed learning with manipulatives and continued to use them and play with them after the initial concepts were learned. Students above age sixteen acknowledged their role in the learning process, but after the desired results were achieved, they had no interest in the extended use of the manipulative. Most reluctant to use manipulatives were those aged twelve through sixteen for various different reasons: neither sex wanted to play with anything too juvenile, but the boys were even more disinclined to participate when the manipulative had any feminine qualities.

The use of manipulatives to introduce a topic is only meant to be for the initial instructional period which can vary depending on the manipulative being used. It can be time consuming, so it reasons students should eventually move past this towards symbolic manipulation. Manipulatives that model and reinforce algebraic concepts aid students while they move from a concrete operational level to an abstract symbolic level (Balka, 1995; Deubel, 2010; Lindroth, 2005). It may be difficult to extract the required knowledge from one set of experiences. A variety of activities may be necessary for students to make the bridge from concrete to abstract (Dienes, 1967). The positive aspect to this is that if a student ever forgets a particular formula or how to carry out a particular abstract symbol manipulation, they can always fall back on their initial discovery method to try to work out a problem (Curriculum, Technology, & Education Reform, 2008).

Dienes (1967) based his early theory of mathematics learning on the work of Piaget, Bruner and Bartelett as well his own experimental results. Dienes (n.d.) eventually developed his theory of learning mathematics to include six stages. He calls stage one the free play stage; he feels this should be the beginning of all learning. This is the time for trial and error where students become familiar with the situation that confronts them. During the first stage regularities begin to appear which lend itself to the rules involved in stage two: learning to play by the rules. He suggests that students play a number of mathematical games with several similar rule structures that use different materials so that the common theme or underlying concept may become more transparent to the students. Stage three, the comparison stage, uses a varied approach also intended to bring the students to the realization that the manipulatives used for playing the games are less important than the rule structure being developed. Discussion is encouraged at this stage. Students will be encouraged to take the first step from the concrete to the abstract through these discussions. Stage four is the representation stage. Students are given a method in which to represent the abstract concept such as a diagram, a table, a coordinate plane or some other type of mean of expression. Each one of the games is then applied to this representation demonstrating the common thread from which they stem. Symbolization is applied during stage five. Properties of the games are discussed and terminology is applied to those properties. Formalization takes place in the final stage where axioms, theorems and proofs are then related to the previous five steps. Dienes (1967) states that the analytical way of investigating a situation was a rather mature way of thinking and seldom occurred before the age of twelve.

In using manipulatives in the classroom, students use their immediate knowledge of manipulating concrete objects to make observations on how those objects work. This discovery approach will, in hope, allow students to come to a conclusion and be able to grasp the parallel abstract mathematical concept (Curriculum, Technology, & Education Reform, 2008). Beyond the initial instructional period where students need to be encouraged to explore and become familiar with the tools—whether concrete or virtual—that they have been given, students should be given specific direction with a specific goal or goals to achieve (Lindroth, 2005), or a game as Dienes (n.d.) calls it.

Before, throughout, and following the exploration period, there should be open dialogue between teacher and students. More importantly this dialogue should be among students. In a traditional classroom setting, most interaction occurs between the teacher and the students, but with the use of manipulatives, there is a lot more student-to-student communication (Trotter, 2007). This interaction allows the students to get clarification, feedback, and insight into other students' thought processes (Burns, 2004; NCTM, 2000; Raymond & Leinenbach, 2000). It is important that the instructor make time for this communication when working with manipulatives (Waite-Stupiansky & Stupiansky, 1998) as working with them makes students eager to participate in class discussions (Balka, 1983).

Waite-Stupiansky and Stupiansky (1998) present some guidelines for the use of manipulatives so the students are actively engaged instead of simply going through the motions: have the students dialogue in some way—whether written or oral—about their experience with the activity; ask probing questions in order to ignite students' thinking; use hands-on-tools in lessons in appropriate places; have the students write or diagram as a way to keep track of their progress on an activity; and the instructor should be sure to evaluate each activity to be sure that students were mentally and physically engaged and learning took place.

The instructor should be familiar with the set-up and specific instructions before class as to optimize the students' time with manipulatives (Lindroth, 2005). Instructor organization is a key piece to student success (Stewart, 2003). For virtual manipulatives, is helpful to have the computers set up and the specific site or applet that will be used loaded on all of the machines or else have a short cut created that the students may access. Oftentimes it is helpful for the instructor to create a handout with some general guidelines where the students can record their progress and some possible conclusions, thus making it easier for the instructor to monitor their progress (Lindroth, 2005).

For many manipulatives, the instructor can give some initial direction, but should then leave students to draw their own conclusions. If a student is led down the wrong path, the instructor can guide him or her back on track with a series of hints or questions allowing them to find their own mistake, rather than just telling them (Curriculum, Technology, & Education Reform, 2008). Also, when necessary, the instructor can ask a series of probing questions to try to focus a student's thinking (Waite-Stupiansky & Stupiansky, 1998). Even when a student produces a correct response or result, the instructor should look for more, asking for clarification or further explanation. This line of questioning will confirm a student's understanding in case they had a lucky guess (Dienes, 1967; NCTM, 2000). Following this, the instructor should also address the class to see if a student used a different method or got a different answer (Burns, 2004). Incorrect solutions should also be discussed, but the instructor needs to make it clear that it is the ideas that are at stake, not the students who suggested them (NCTM, 2000).

According to Dienes (1967) and NCTM (2000), the instructor should not take on the authoritarian role but one of a counselor that guides the students. An annoyed comment or a disapproving tone of voice may be a hindrance in the learning of the student in this type of a learning situation.

Using manipulatives as another form of direct instruction is one mistake made by instructors in the implementation of manipulatives. Students are told exactly what to do with the manipulatives, what they represent, and what they should find as a result of the activity. Students are told step-by-step instructions and are not allowed to make any of their own discoveries or conclusions. Thus, in following through the motions, they are not forming any of their own knowledge or making any connections (Curriculum, Technology, & Education Reform, 2008). If mathematical situations are not learned as generalities, their principles will not become familiar to students and they then will not be able to recognize similar situations (Dienes, 1967; Jones, 1986).

A grave assumption that instructors sometimes make is that they view the link from the manipulative to the abstract concept as transparent when it is not at all clear to the students (Clements, 1999). This correlates to another mistake made when instructors do not allow enough time for students to make the necessary connections and they progress to the symbolic manipulation too soon (Dienes, 1967; Jones, 1986).

When finished with an activity, the instructor should evaluate it and try to refine the process (NCTM, 2000). Waite-Stupiansky and Stupiansky (1998) give us two things to look for: were students engaged mentally as well as physically and was there a purpose or were students just generating solutions?

There are often multiple approaches for describing something. Each has its own advantage, regardless if it is the desired solution (NCTM, 2000; Resnick et al., 1998). Burns (2004) reminds instructors that confusion and partial understanding are part of the natural learning process. Errors are opportunities for learning, and the classroom environment should support children taking risks without the fear of failure or embarrassment.

When using virtual manipulatives in the classroom, it is important that the parents understand the value in it. Many parents see their children just playing on the computer versus working on a valuable homework assignment. Sending a letter home explaining the specifics of the manipulatives and how they reinforce the concepts being taught in the classroom can go a long way. The instructor can also have a sheet that the parents can sign to show that their child has been working on the assigned manipulatives (Lindroth, 2005; Stewart, 2003).

Research on Manipulatives

Through the use of manipulatives, Balka (1995) envisioned a classroom environment where the dull image of algebra would yield to an algebra that is filled with student involvement as they begin to better grasp the abstract mathematical concepts.

There are two ways in which the effectiveness of the use of manipulatives can be measured: affect and achievement. How students perceive mathematics is known as affect, while achievement is the noticeable difference between mathematics learned and able to be completed by students before and after a unit (Curriculum, Technology, & Education Reform, 2008).

The use of manipulatives in the classroom has been found to have a positive effect on affect (Balka, 1983; Clements, 1999; Curriculum, Technology, & Education Reform, 2008; Garrity, 1998; Hinzman; 1997; Raymond & Leinenbach, 2000). Some studies on the use of manipulatives in the classroom have not yielded significant increases in student grades but have shown small academic gains in addition to an increased attitude towards mathematics. Hinzman (1997) did an eighteen week study on eighth grade pre-algebra students. Her research did not show any dramatic gains in student grades, but her conclusions in the support of manipulative use came in response to student attitude. She discovered that students' overall feelings, as well as their attitude toward mathematics, greatly improved. Garrity (1998) did a study with her secondary Geometry courses. Her findings paralleled those of Hinzman in that the gains in positive attitude were significant.

Many studies have shown that positive student achievement results from the use of manipulatives in the mathematics classroom (Haas, 2005; Hinzman, 1997; Raymond & Leinenbach, 2000; Resnick et al., 1998; Zuckerman et al., 2005). Positive results have also been demonstrated in studies done with special education students (Balka, 1983; Maccini & Ruhl, 2000). If the contrary occurs, and the use of manipulatives results in a negative experience, it is usually due to the instructor's misunderstanding of the use of the manipulative or the student is lacking in a foundational concept that should have been learned previously (Curriculum, Technology, & Education Reform, 2008, McClung, 1998). If the latter is an issue, Dienes (1967) reminds us that it is necessary for the instructor to introduce the student to further experiences to strengthen the concepts that are being learned. No matter the age of the child, the use of manipulatives often causes the validation of students' informal knowledge of mathematics by the instructor. Compared to lecture, most students are in favor of the use of manipulatives, as many of them are fun and interesting and sometimes even game-like (Raymond & Leinenbach, 2000). Also, most students are able to find some level of success which is always an encouragement (Curriculum, Technology, & Education Reform, 2008).

Raymond and Leinenbach (2000) spent two years on collaborative action research on the learning and teaching of algebra; the initial results supported the use of manipulatives in the classroom. In the initial year of the study, Leinenbach used both the traditional textbook approach for a certain time period and then used a manipulative approach to algebra with the same group of students. Overall, student scores were higher during the periods of time that manipulatives were used, with some students showing significant increases. In addition, the use of manipulatives increased the positive attitude towards algebra. Ninety-one percent of her students preferred the use of manipulatives when compared to the traditional method of instruction. The following year, Leinenbach was only able to do follow up on approximately sixteen percent of her students. Due to such a small population the long term affects were not able to be determined conclusively.

Researchers from SRI International, University of Massachusetts at Darmouth, University of Texas at Austin, Virginia Tech, and the Charles A. Dana Center at the University of Texas at Austin conducted three large-scale experiments to determine if a technology-enhanced curriculum would increase student learning in mathematics at the middle school level (Center for Technology in Learning, 2007). Interactive software-based curriculum SimCalc, created at Darmouth, is a type of virtual manipulative that is meant to enhance the strand of mathematics relating to much of algebra and leading to calculus (Roschelle et al., 2009). It is a computer program that can be utilized with or without the graphing calculator (Trotter, 2007). According to Roschelle et al. (2009), findings held true across all studies: students learned advanced mathematical concepts without sacrificing increased state test scores.

Haas (2005) conducted a meta-analysis reviewing and integrating research that was done between 1980 and 2002 on teaching methods for secondary-level algebra. The research defined six specific teaching methods types and ranked them according to their effect. Thirty-five studies were chosen from over fourteen hundred studies. His findings ranked from the method with the largest positive effect to the smallest effect are as follows: direct instruction; problem-based learning; manipulatives, models, and multiple representations; cooperative learning; communication and study skills; and technology-aided instruction. In his findings, all six methods produced some form of positive effect, but direct instruction had the greatest overall effect resulting in a twenty-one percent gain in percentile when compared with the control group. Haas did make a point to note that this type of teaching method may encompass all of the others. The use of manipulatives ranked third showing a percentile gain of 15% in student achievement. Haas stated that this method was best utilized to enhance other teaching methods for algebra by providing opportunities for hands on learning as well as to communicate to others. He concluded that teachers should integrate the use of all these research-based teaching methods in order to optimize instruction.

Identifying Desirable Manipulatives

When dealing with the creation of manipulatives, Eisenberg & DiBiase (1996) recommended the following be kept in mind: the age of the students the manipulatives will be used with, as they must not appear too childlike when being used with teenagers; the manner in which the students will be observed by their peers when using the manipulatives; and also to create gender neutral manipulatives. NCTM (2000) recommends the use of instructional materials that are intentionally designed to weave together different content strands. Something else to keep in mind is that if the activity involved is fun and exciting, students will not ask if it is useful (Dienes, 1967). The best activities meet the needs of all students at all levels of interest and experience. Instructors must look for or create activities that allow students to seek their own level and that lend themselves to natural extensions (Burns, 2004; NCTM, 2000).

Dienes (1967) recommends diversifying as many variables as possible in order to provide the highest opportunity for concept growth. In his Perceptual Variability Principle, in order to present the largest opportunity for student success in understanding an abstract concept, Dienes (1967) suggests the concept in question should be presented in as many different equivalent forms as possible. He describes a situation where students can construct a parallelogram on paper, create one out of two wooden triangles, trace one with pegs on a pegboard, and also find their shape in real life. By examining several different representations, the students' perception of the abstract mathematical concept is enhanced. Also in his Mathematical Variability Principle, Dienes (1967) communicates how general mathematical concepts should be learned by tasks that diversify the components as much as possible. By using the same manipulatives to introduce new concepts, students do not have to spend time learning how to use a new manipulative, and instead, they can spend the extra time practicing the new concept.

Chapter 3: Discussion of Research

My School

Bemidji High School is attended by students with all levels of mathematical ability. BHS follows a traditional mathematics curriculum. I have taught every level except calculus. Each level of mathematics presents individual challenges to the teacher. Students entering BHS start their mathematics curriculum at one of five different levels: Resource Mathematics, Pre-algebra, Algebra 1A, Algebra 1B, or Accelerated Algebra 2. To create a detailed picture of our school, I will present certain percentages. All of the percentages contained in this section were calculated from data collected in July of 2010 from Bemidji High School records and are used with permission.

Accelerated Algebra 2 students have already passed a traditional first year algebra course. Typically highly motivated, these students have been exposed to a variety of mathematics and have a firm grasp on many abstract concepts. Instructors need to be exceptionally prepared as students will absorb everything presented. Twenty-two percent of the 2010-2011 freshmen are taking Accelerated Algebra 2.

In contrast, there are the low level students which consist of two groups: those who receive special education services and those who do not. Sixteen percent of the 2010-2011 BHS freshmen will receive special education services. Half of these students will get their mathematics credits from a resource classroom that is taught by special education instructor and will not be taught in the mainstream classroom. The other half of those receiving special education services can be found throughout all levels of mathematics.

The low level students who do not receive special education services start in our Pre-algebra course. These students, about 8% of the 2010-2011 freshmen, are there for a gamut of reasons: some slipped through the cracks, some have been elevated through the system with social promotion, some transfer between schools and have not been held accountable, and then there are a few that do actually struggle in learning mathematics. The students entering ninth grade have typically seen the content contained in our Pre-algebra course prior to high school which allows the instructor more flexibility with respect to time. You can afford to take extra time on a lesson or to review things like arithmetic, fractions or other areas where the students may not have developed strong skills as was recommended by Dienes (1967). Unfortunately, this is usually where your students with behavior problems end up. It is a challenge to keep the students engaged every day for our ninety minute class period.

Finally, we have the students who start in Algebra 1. The mathematics department has broken this into two classes: Algebra 1A and Algebra 1B. Algebra 1A includes the topics:

- Expressions, Equations and Functions;
- Properties of Real Numbers;
- Solving Linear Equations;
- Graphing Linear Equations and Functions;
- Writing Linear Equations;
- Solving and Graphing Linear Inequalities; and
- Systems of Equations and Inequalities.

The Algebra 1B course includes the topics:

- Exponents and Exponential Functions;
- Polynomials and Factoring;
- Quadratic Equations and Functions;
- Radicals and Geometry Connections;
- Rational Equations and Functions; and
- Probability and Data Analysis.

During the 2006-2007 school year, 37% of the freshmen started in Algebra 1A, with only 5% of the students starting in Algebra 1B. Due to changes in state requirements, many of our students are taking Algebra in the eighth grade. This has caused the number of students starting in both Algebra 1A and Algebra 1B to increase. In the 2010-2011 school year, 45% of the freshmen are starting in Algebra 1A while 17% are starting in Algebra 1B.

The students that begin in our Algebra 1A courses are the ones who concern me the most. These are the students that I want to help. I believe that it is necessary to try to give them the largest possible mathematical foundation on which to build. In the past, my teaching methods have consisted of lecture with other teaching methods—group work, technology, and manipulatives—incorporated on a limited basis. I wanted to change something in my instruction to try to get these students interested. Some of the students failed the course due to attendance issues. Maybe if the class were more enjoyable, I could retain more of these students. For those that did pass, gaining a commanding understanding of the material in the course is an issue. They do the bare minimum to pass the course. Little retention is common, and students still possess their negative attitudes towards mathematics. This similar scenario in traditional mathematics was documented by Dienes (1967) over fifty years ago. Unfortunately, as reported by NCTM (2000), disengagement is all too often reinforced in today's society by the attitudes and actions of sources such as parents, other authority figures, and the media. The message "that not everyone is expected to be successful in mathematics" is heard loud and clear, and therefore it is acceptable in society—almost the norm—to dislike mathematics.

Only three mathematics classes are required at Bemidji High School. Some students will only take Algebra 1A, Algebra 1B, and Geometry and will not take another mathematics class in their high school career, while a few will progress all the way through AP Calculus. Twenty-one percent of the class of 2010 took the minimum required mathematics courses with an additional 12%—mostly special education students—who did not even reach this level. Fifteen percent of the class of 2010 took AP Calculus during their senior year. Ninety percent of the students that successfully completed a full year of AP Calculus during the 2009-2010 school year began high school in the accelerated track. The other 10% of the students who successfully completed a full year of AP Calculus took Algebra 1A as a freshman. Although he does not keep detailed records, AP Calculus instructor T. Hewitt thought 10% seemed slightly higher than normal (personal communication, July 10, 2010). Regardless of which path they choose to take, I feel it is necessary to give them as many tools as possible in order for them to have a successful mathematics future. Similarly, NCTM (2000) advocates that all students have access to a common foundation of challenging mathematics whether those students will enter the workplace after high school or pursue further studies in mathematics and science.

Forty-one point two percent of the BHS class of 2010 did not meet proficiency on the MCA II mathematics exam compared to the public school state average of 42.7%. An additional 16.8% of the students who were not proficient on the MCA II exam did score high enough to pass the embedded GRAD exam. This means a total of 42% of the students did not pass the GRAD exam. Of these students that did not meet proficiency, 14% took resource mathematics, 28% started high school in the Pre-algebra course, 37% started in Algebra 1A, 3% started in Algebra 1B, and 4% started in Accelerated Algebra 2. The other 14% who did not meet proficiency were transfer students. Those 42% who did not meet proficiency were required to take a mathematics class their senior year or to do some type of remediation before they retook the GRAD exam. Of those that did not pass their junior year, half never passed the exam their senior year, although 20% of these students did not retake the exam two more times as was required. My hope is that, as I continue to grow, I will be able to share my findings with those in my department, as by 2014, the government's goal is 100% proficiency for all students. If we are to continue to increase our student achievement, we are going to have to continue to grow as a department. This will be challenging. Dienes (1967) observed years ago that teachers, especially those who are steeped in a traditional method of handling both the subject and the methodology of teaching, are very difficult to move. Regardless of the difficulty, teachers must challenge and hold high expectations for all their students regardless of their ability level as recommended by NCTM (2000).

My Experiences

Since I began teaching, my instruction methods have continued to evolve. I feel that I have done a sufficient job in my classroom, but as each year goes by, I have wanted to do a better job. I have been changing what I do in the classroom, little-by-little, trying to develop additional ways to deliver the material to the students in order to increase understanding and increase positive attitudes. I am finding that through the use of manipulatives and classroom activities, students are more engaged, students have a more positive attitude toward mathematics, and some students seem to be able to retain more information long term.

At the beginning of my teaching career, I was the youngest in a department of twelve mathematics teachers. Lecturing from the book was how the instructors delivered the lessons. This, of course, seemed to work well for the accelerated courses but not always so for the lower algebra students.

I tried to implement a few hands-on activities into my curriculum in my early years of instruction, but they never seemed to go well. I once tried an activity using black and red beans to represent negative and positive numbers in my pre-algebra class. It was a nightmare. I tried to teach them how to use the manipulatives to do the problems instead of teaching them what the manipulatives represented and allowing them to make their own discoveries and come to their own conclusions. I also did not have any handouts for the students to give them a goal or direction. It was not long before students were playing with the beans and they were flying across the room. Beans are also not a great manipulative as they split and make a mess all over the floor.

When I attended high school, we were not allowed to use calculators regularly in our mathematics courses. We were able to use a TI-85 on a very limited basis in trigonometry and in calculus. I used it more often during my college career, but it was not uncommon to have a calculator and a non-calculator version of a test. When I started teaching at BHS, some teachers were incorporating the TI-82s and TI-83s into their curriculum. The calculator was becoming an integrated part of the classroom. Unfortunately, I went too far as I began to use the calculator as a replacement for basic understanding and intuitions. I taught them step-by-step instructions on how to arrive at a solution. According to NCTM (2000), learning without understanding has been a common outcome of school mathematics instruction. At least I was aware what I was doing was not working. Many students were able to go through the motions and arrive at a solution, yet they had no idea what it meant. They had no idea what a problem was telling them, especially when placed in any type of a context. I have become more successful in teaching with the calculator in recent years. I have learned to use the calculator from the onset, but first giving students the foundation they need in order to better understand what the calculator is showing them instead of just some meaningless answer on their screen.

In my effort to reach Algebra 1A students, I began to incorporate all levels of group work: working in small groups, doing class work on the board, and group tests. This did have some positive results. Clements (1999), Haas (2005) and Jones (1986) recommended a balanced approach to instruction that included the use of manipulatives. This was reassuring to me. I have been using direct instruction, group work, and calculator methods, but it was still not enough.

Recently I have tried to incorporate activities and the use of manipulatives. I have no formal training with this method of instruction other than what I learned in my college courses. I am out of my comfort zone when it comes to the use of manipulatives and incorporating them into my classroom, and I have found it challenging. I will be the first to admit that I have a lot to learn as this latest addition to my teaching methods is in its infant stages, but I feel I am headed in the right direction for both myself and my students. Through the implementation of manipulatives I hope to further develop my Algebra 1A course as well as enrich my other classes.

Use of Manipulaitves

There is a plethora of research on the use of manipulatives in the classroom. In my research I discovered that there are three different types of manipulatives: concrete, digital, and virtual. As discussed by Balka (1983), Burns (2004), and Lindroth (2005), there are a lot of different types of concrete manipulatives for all levels of mathematics which aid students in obtaining a concrete understanding of abstract mathematical concepts. The definition of the manipulative has developed over the last couple of decades to include the advances of technology and the creation of digital and virtual manipulatives as explained by Craighead (2004), Eisenberg & DiBiase (1996), Eremin (2007), Lindroth (2005), Marshall (2007), Resnick et al. (1998), and Zuckerman et al. (2005). Concrete, digital and virtual manipulatives exist for all levels of mathematics, although as Resnick et al. (1998) point out, digital and virtual manipulatives are at times better for the exploration of advanced abstract concepts.

I concur with Chappell & Strutchens (2001) that the use of manipulatives is a time consuming instruction method that is not always fully understood. I am now putting in the time to learn about them and feel the benefits will outweigh any extra time it takes me in research or preparation. Clements (1999), Duebel (2010), and Raymond & Leinenbach (2000) voiced their concern that students may become reliant on the manipulatives and be unable to make the bridge to the corresponding abstract concepts. This may be the case, but I have seen students struggle with learning the abstract concepts without the use of manipulatives. I feel introducing the concept using a manipulative and eventually trying to make the connection to the abstract concept is worth the chance. In the past, I have avoided manipulatives for other reasons as well: lack of funds to purchase as mentioned by Stewart (2003) and not having adequate numbers to meet classroom needs as pointed out by Lindroth (2005). I have also avoided the use of manipulatives due to my lack of knowledge and experience. As mentioned previously, I have not had any professional development in this area, and my professional training is limited to brief exposures during my undergraduate and graduate studies.

I found Eisenberg & DiBiase's (1996) observations about the use of manipulatives and the ages of the students interesting. I could not agree more with their observations about teenagers. Even with my limited use of manipulatives in the classroom, I have observed the reluctance of the ninth and tenth grade students to use them, while I have also observed the eleventh and twelfth graders who will quickly use a manipulative ready to move on to the abstract application. I have also witnessed the reluctance of the males to participate—not just in the ninth and tenth grades as they reported—but in the eleventh and twelfth grades as well. This is not always the case, as Raymond & Leinenbach's (2000) results indicated that 91% of students preferred the use of manipulatives compared to the textbook.

Dienes (1967) writes that it may be difficult for students to abstract from one set of experiences. I think that when teaching in the Algebra 1 classroom, if students are struggling after the initial activities, other experiences may be offered to those students on a small group or a one-on-one basis as necessary. This is something that I will have to deal with on a topic-by-topic basis though, as there may be certain areas where it is beneficial to the entire class to provide several initial experiences.

I liked Dienes' (n.d.) six stages of learning: free play; learning to play by the rules; comparison and discussion; representation; symbolization; and formalization. In my interpretation, these six stages seem to be something that could happen over short or long periods of time. There was no other information on Dienes' web page about his six stages of learning. It seems that in a traditional algebra setting, that the first four stages are skipped and instruction begins with the introduction of symbolization, yet students do not have any experiences to draw from or to relate to. This is a likely source in the confusion and lack of understanding among traditional algebra students.

Balka (1983), Burns (2004), NCTM (2000), Raymond & Leinenback (2000), Trotter (2007), and Waite-Stupiansky & Stupiansky (1998) all had similar ideas about communication in the classroom. I agree communication is an important piece of any curriculum, not only between the instructor and the students but even more so between the students. Often students are able to help each other understand as they are at the same level intellectually compared to the instructor who knows where everything is leading, mathematically speaking. The instructor needs to make time for this during class. I have also found that there are instances when you do not have time that good discussions have come up. NCTM (2000) tells us that this is something instructors need to be prepared for. In my experience, it is worthwhile to make the extra time as the students will benefit from it.

Lindroth (2005) recommends when using manipulatives, students should be given a specific goal. Also, Waite-Stupiansky & Stupiansky (1998) suggested that hands on tools are used where appropriate and either written or oral student dialogue is incorporated so students are not just going through the motions. These seem like reasonable guidelines to follow. Lindroth (2005) wrote that instructors should be familiar with the set-up and specific instructions before class in order to optimize students' time with the manipulatives. This seems like common sense to me as it is important for instructors to be prepared regardless of their instructional methods. Also Lindroth (2005) recommends that the instructor create a handout with some general guidelines where students can record their progress and record possible conclusions. I feel this is a must, especially when dealing with immature high school students—if some of them do not have specific direction, they will not do anything. Dienes' (1967) recommendation to continue questioning students—even after the correct solution is given—is something I now plan to do. Often, I was satisfied when a correct answer was reached and would clarify and move on. He suggests that you continue questioning in order to clarify understanding. I have done this intermittently in the past, but I think it would be good for all students, regardless of their level of understanding, if I were to make this common practice.

Dienes (1967) and NCTM (2000) believe that the instructor should take on the role of the counselor instead of an authoritarian role. An annoyed comment or disapproving tone of voice may be a hindrance in the learning of the student with this type of instruction. In my opinion, regardless of the instruction method, it is detrimental to learning when students are reprimanded for the inability to recall a previously learned concept or for not understanding the mathematics involved. The effects of one incident can stay with a child for years. I have always tried to avoid doing this, as I still carry with me some of my own negative experiences from when I was a high school student. There are a few times in my career where I did not even realize at the time that what I was saying was being taken negatively until I saw the reaction on the student's face. These are situations that I as instructor do not easily forget either. For the past several years, I have been sought out by students who would like more clarification as they are afraid to approach their own instructor in fear of their reaction.

Clements (1999) revealed a mistake I believe will be difficult to avoid until I become more experienced in the use of manipulatives. He noted how instructors may view the link from the manipulative to the abstract concept as transparent when it is not yet transparent to the students. Another mistake, as revealed by both Dienes (1967) and Jones (1986), is the failure of instructors to not allow enough time for students to make the necessary connections before progressing to the correlating symbolic manipulation. I hope to avoid both of these mistakes through bridging activities, whether by finding premade ones or creating my own to tailor fit the lesson. The practice of bridging concrete or real world examples to the related abstract concepts is also what is recommended by NCTM (2000).

Lindroth (2005) and Stewart (2003) recommend keeping the parents informed on the use of manipulatives. I plan to include a section in my information sheet at the beginning of the course explaining what we will be doing in class. I send this sheet home with students during the first week of class and have them return it with a parent signature. At the high school level, parents still want to be informed of what is going on in their child's education, even though at this age, communication between the child and the parents often begins to wane due to the child's growing independence.

Balka (1983), Clements (1999), Garrity (1998), Hinzman (1997), and Raymond & Leinenbach (2000) report the use of manipulatives increases positive student attitude regardless of the increase of student achievement. I can see how this would be a positive outcome as students are able to do something during class compared to lecture where they sit and listen which can quickly become boring for many students. I believe that if this is the only result that it is still worth the incorporation of manipulatives into the classroom.

Increase in achievement is often a result of the use of manipulatives in the classroom. My findings included both small scale (Hinzman, 1997; Raymond & Leinenbach, 2000; Zuckerman et al., 2005) and large scale (Haas, 2005; Roschelle et al., 2010) studies that displayed at least some increase in achievement. I was not searching for negative results in regard to the use of manipulatives, but in looking for research on Algeblocks, I found a study by McClung (1998) that did not result in increased achievement. McClung (1998) did discuss though how his results could have stemmed from his inexperience instructing with the manipulative. Curriculum, Technology, & Education Reform (2008) explains how if the use of manipulatives results in a negative experience, it is usually due to the instructor's misunderstanding of the use of the manipulative or the student's lacking of a foundational concept that should have been learned previously.

Review of Manipulatives

My immediate plans are to incorporate the use of Algebra tiles, virtual manipulatives, and specific activities that I have discovered, into my Algebra 1A class. I am also pursuing the purchase and implementation of Algeblocks and a SMART board for use in my classroom.

Larson, Boswell, Kanold, & Stiff (2007) incorporate the use of Algebra tiles intermittently in their traditional Algebra 1 textbook which is currently used at BHS. The lessons are small activities, and through my own experiences, these activities are very limited. An Algebra tiles investigation booklet (Larson, 2001) was written in correspondence with a previous version of the text book. These are lessons that I will be able to use, but they are not inclusive of the topics for which I would like to use them. Integer operations are excluded from the lessons, and even though solving two-step equations is included, solving one-step equations is not. An additional book I found (Hall, 1994) will be a great supplement to Larson's (2001). In addition to the topics included by Larson, Hall also includes all the introductory lessons involving integer operations that I was searching for. In order to ensure student success I am creating my own bridging exercises. These will include the drawing of the model, describing the model using words, and finally the corresponding symbolic manipulation of the model.

I discovered a virtual manipulative that models linear equation solving using Algebra tiles (n.d.). It allows the user to model one and two step equations with variables on one or both sides. It shows the abstract notation in correlation with the Algebra tiles. Users can input their own problem or they can choose to use premade problems. After learning how to solve linear equations using Algebra tiles, I would like to allow students to learn how to use this virtual manipulative in class. By using it in class I can ensure that all students understand how to properly use it. Students should then be able to use this at home to practice as well as check their answers. The only thing I do not like about this virtual manipulative is that it does not tell the user what he or she did wrong when solving an equation, only that they got the incorrect answer.

In my opinion, I have underutilized virtual manipulatives in my classroom. I have access to computers, and I believe this is something that I need to incorporate into my lessons. Even though I have not used digital manipulatives in the classroom, I can see Marshall's (2007) reasoning why he feels virtual manipulatives have an advantage over digital manipulatives in terms of storage space and set up time. I also agree with Clements (1999) on several other advantages of virtual manipulatives: some easily lend themselves to more extensions; some allow students to pick up where they left off as opposed to starting over again each class period; and some give you instant feedback or are able to replay the process allowing for further exploration. One of the greatest advantages Clements (1999) points out is the ability for some virtual manipulatives to link the concrete experience simultaneously to the abstract symbolic representation.

Deubel (2010) warns of problems to consider when dealing with virtual manipulatives. He states that instructors must be prepared as you are relying on properly working computers, internet connections, and the sites you are accessing. I have taught Web Design using html for ten years so I am well aware of the issues that instructors have to tackle when using computers and am ready to deal with them.

There are numerous virtual manipulatives available online. You can look for them using a search engine, but there are many sites available that list and categorize the applets in order to save instructors time. One of these lists is the *National Library of Virtual Manipulatives* (n.d.). The National Library of Virtual Manipulatives is a National Science Foundation supported project conducted by Utah State University that began in

1999 in which a library of virtual manipulatives encompassing K-12 mathematics instruction was created. It is broken down by both grade level and content area which includes Number & Operations, Algebra, Geometry, Measurement, and Data Analysis & Probability. Two examples of applets that I plan to use in my Algebra 1A course are "Point Plotter" and "Line Plotter." Plotting points and graphing lines is a concept that many students "think" they understand. Usually most students do understand the concepts, but tend to make little mistakes. These applets will allow students to practice their skills giving them instant feedback as both applets check the given solutions.

A second web site which includes over one hundred online activities is Illuminations: Activities (n.d.). These activities are broken down by grade band as well. Two activities that I plan to implement into my Algebra 1A curriculum are "Compound Interest Simulator" and "Trout Population Calculator." These activities will fit in well when students are learning about exponential growth and decay.

A third collection, Project Interactivate (n.d.), was sponsored by Shodor Education Foundation. This collection of 100 virtual manipulatives was created for exploration in science and mathematics. The site also includes 70 NCTM standards based lesson plans and a dictionary of mathematics terms used throughout the site. The activities are again broken down by content area and grade level. Two applets from this collection that I plan to use in my Algebra 1A curriculum are "Function Machine" and "Linear Function Machine." Both applets investigate linear functions where students have to determine the algebraic form when given inputs and outputs. The first applet includes basic functions that involve only one operation. In the second applet students must find both the slope and the intercept. Both these applets check student solutions so they will be given immediate feedback. They also "keep score" so that the instructor can easily monitor student progress.

There are a multitude of books available that contain premade activities for use in the classroom at all levels of mathematics. I have reviewed some of them, and the books that I discuss are the ones that I have found useful and plan to implement into my Algebra 1A curriculum.

The first book is *Exploring Algebra and Pre-algebra with Manipulatives* (Balka, 1995). This book contains hard copies of games and manipulatives that can be duplicated for use in my classroom. Some of the activities require the use of color chips or other inexpensive manipulatives, but for most of the activities, instructions are given information on how to create your own if necessary. Two activities that I plan to use from this book are "Zero" and "Modeling Number Puzzles." The first activity, "Zero," is designed to practice addition and subtraction of positive and negative numbers. Students toss ten two colored counters—one side represents a negative and one side represents a positive—and record their sum. After ten rounds, students find the overall sum. The player closest to zero wins the game. The second activity, "Modeling Number Puzzles," first requires students to practice basic operations on numbers and then progresses to basic operations on a variable through the use of number puzzles.

The second book where I found useful activities was *Hands on Algebra! Ready-to-Use Games & Activities for Grades 7-12* (Thompson, 1998). These lessons appear to be easily integrated into our traditional Algebra 1 text. Each objective in the book is presented in a series of three lessons or activities. Activity 1 involves physical models in the development of the concept. Activity 2 uses pictorial or graphic models to help students bridge from the concrete to the abstract. Students then draw conclusions and make generalizations from these activities. Activity 3 allows students to further explore these generalizations. The manipulatives involved, the instructor's role, any student worksheets, and the approximate time that the activity should take are all included in the book. Looking at the activities, the manipulatives involved are all things that can be created out of inexpensive items. There are templates included for out of the ordinary things that are used. The objectives of a series of activities I plan to use are to "investigate linear or arithmetic sequences, using figurate numbers" and to "find an expression for the N-th term." In this set of three activities patterns are built, drawn and then in the final activity they are graphed on a coordinate plane. I have come across numerous activities that build patters in order to introduce linear functions, but this is the first that also attempts to display the patters graphically.

The other books from which I plan to draw activities for my Algebra 1A curriculum are part of the Navigation Series from NCTM. *Navigating through Algebra in Grades 3-5*, (Cuevas & Yeatts, 2001), includes activities that introduce patters, variables and equations, and functions. Even though NCTM's target audience is grades 3-5, I will be able to incorporate some of these activities when introducing a topic. Two activities that I plan to use are "Watch Them Grow" and "Squares Cubed." "Watch Them Grow" is an introductory activity used to build linear functions. Students build patterns and find the rule for their pattern. In "Squares Cubed" students investigate the relationship between perimeter, area, and volume and attempt to describe the relationships and the patterns that they discover. This activity can be used in relation to function building as well. I also plan to use activities from *Navigating through Algebra* *in Grades 6-8* (Friel, Rachlin & Doyle, 2001). In this book both “Exploring Houses” and “Building with Toothpicks” take another look into patterns that develop linear relationships.

Algeblocks are a concrete manipulative that I have not yet had access to but have read about. Unfortunately McClung’s research showed that students using the traditional method of lecture, homework and in class worksheets outperformed the students taught using the manipulatives. It was a small study, and regardless of his research, I have heard from other colleagues the benefits of Algeblocks. M. White, a special education teacher at Bemidji High School, told me that some of the special education teachers at BHS have been using Algeblocks in the resource mathematics classes with successful results (M. White, personal communication, June 2, 2010). Strom (2009) has used Algeblocks in her classroom to instruct many different concepts: integer operations, simplifying expression, solving equations, solving systems of equations, and composition of functions. She has seen positive results in her classroom after using these manipulatives. At the end of the 2009-2010 school year, I was able to order some Algeblocks. It is not a full classroom set, but I plan to use them with students who need more practice as Dienes (1967) encourages. From this I will be able to form my own opinions.

I recently got an LCD projector in my classroom, and I have a laptop for use in my room as well. These tools will make it easy to implement virtual manipulatives into my classroom for demonstrational purposes. In recent years I have spoken to colleagues in nearby school districts that have begun to incorporate the use of SMART boards in their mathematics classrooms. They feel that these are a valuable tool in the classroom, and they cannot imagine teaching without one. Our district has purchased some of these through grants. Although I applied, I did not receive one at that point in time. I am in the process of trying to acquire a SMART board for use in my classroom through the district technology specialist and my high school principal. Similar to the online collections of virtual manipulatives, there are online collections of SMART board lessons. No one in the mathematics department at BHS has a SMART board, but four teachers at the middle school have access to one. If I am able to get one, I will be required to go to professional training, but I hope to also utilize the experienced instructors at the middle school for references.

Chapter 4: Conclusion

What are the Different Types of Manipulatives?

Mathematical manipulatives are objects that are used to aid a student in learning a mathematical concept by manipulating it. Manipulatives can serve several different purposes: to introduce and develop new concepts, to present a particular problem, and to be used as a problem solving tool. In summary, they help to give students a concrete understanding of abstract mathematical concepts.

There are currently three different categories of manipulatives: concrete, digital, and virtual. Concrete manipulatives are something that a student can hold and manipulate. They serve two purposes: to model real world things, objects and physical structures and to encourage exploration to make abstract manipulations easier. These manipulatives can be everyday items such as cards, money, dice, and toothpicks, or they can be premade, store bought items such as Algebra tiles, Algeblocks, Geoboards, and Base Ten Blocks.

Digital manipulatives are concrete manipulatives with embedded computational power and have expanded the range of concepts that were taught through the manipulation of abstract symbols using formal methods and made them available to many more students. Without the incorporated technology, there are many important concepts that are difficult or even impossible to teach using basic concrete manipulatives. Some examples of digital manipulatives are FlowBlocks, SystemBlocks, BitBalls and LEGO/LOGO.

Virtual manipulatives are online computer manipulatives. Similar to concrete manipulatives, virtual manipulatives help students develop an understanding of abstract mathematical symbols. Virtual manipulatives can be found on the internet and include computer models of concrete manipulatives that are readily available for classroom use.

How Should Manipulatives be used?

Manipulatives are primarily used in the development of a concept. They are meant to be used for the initial instructional period which can vary depending on the manipulative involved. It can be time consuming, so it reasons students should eventually move past this, but the manipulatives should be made available for students to use at any time to help them think, reason, and solve problems. Manipulatives should support the instruction of mathematics topics in the curriculum and should be a regular and integral part of instruction. What may be obvious to an instructor may be a stretch for students to recognize. It may be difficult to abstract from one set of experiences. A variety of activities may be necessary for students to make the bridge from concrete to abstract concepts. It makes sense to introduce one material and provide time for in-depth exploration. On the other hand, using a variety of manipulatives allows students to think about a specific concept in different ways.

In using manipulatives in the classroom, students use their immediate knowledge of manipulating concrete objects to make observations on how those objects work. This discovery approach allows students to come to a conclusion to grasp the parallel abstract mathematical concept. During the initial instructional period students need to be encouraged to explore and become familiar with the tools—whether concrete or virtual. Following this, students should be given direction with a specific goal or goals to achieve. Showing the bridge from concrete manipulation to abstract symbolism is essential. Communication among students is an important component when working with manipulatives. This interaction allows the students to get clarification, feedback, and insight into other students' thought processes and a more robust understanding of the mathematics being explored.

It is important that the instructor make time for classroom communication when working with manipulatives. The instructor should be familiar with the set-up and specific instructions before class as to optimize the students' time with manipulatives. Instructors should create a handout with some general guidelines where the students can record their progress and some possible conclusions, thus making it easier for instructors to monitor student progress.

Instructors should be certain to not use manipulatives as another form of direct instruction. For many manipulatives, the instructor can give some initial direction, but should then leave students to draw their own conclusions. Guiding the students down the correct path, focusing students' thinking, as well as checking for student understanding through a series of probing questions is another responsibility of the instructor. Lastly, instructors should be sure to allow enough time for students to make the necessary connections before they progress to the symbolic manipulation.

What is the impact the use of manipulatives has on student achievement in the secondary mathematics classroom?

There are two ways in which the effectiveness of the use of manipulatives can be measured: affect and achievement. How students perceive mathematics is known as affect, while achievement is the noticeable difference between mathematics learned and ability to be completed by students before and after instruction. There is an abundance of research on the use of manipulatives in the classroom. The use of manipulatives in the classroom has been found to have a positive effect on affect (Balka, 1983; Clements, 1999; Garrity, 1998; Hinzman, 1997; Raymond & Leinenbach, 2000). Some studies on the use of manipulatives in the classroom have not yielded significant increases in student grades but have shown an increased attitude towards mathematics. Many studies have shown that positive student achievement results from the use of manipulatives in the mathematics classroom (Haas, 2005; Hinzman, 1997; Raymond & Leinenbach, 2000; Resnick et al., 1998; Zuckerman et al., 2005). Positive results in achievement have also been demonstrated in studies done with special education students (Balka, 1983; Maccini & Ruhl, 2000). If the contrary occurs, and the use of manipulatives results in a negative experience, it is usually due to the instructor's misunderstanding of the use of the manipulative or the student's lacking of a foundational concept that should have been learned previously.

Curriculum, Technology, & Education Reform (2008) explains how if the use of manipulatives results in a negative experience, it is usually due to the instructor's misunderstanding of the use of the manipulative or the student's lacking of a foundational concept that should have been learned previously. McClung's (1998) research in a secondary algebra classroom did not result in increased achievement when comparing lecture with the use of Algeblocks. McClung did discuss how his results could have stemmed from his inexperience instructing with the manipulative. Teachers should integrate the use of as many methods as possible in order to maximize instruction.

What Types of Concrete and Virtual Manipulatives could be used in an Introductory Algebra Classroom with a Traditional Algebra Curriculum?

Algebra tiles, along with various homemade manipulatives, will be the concrete manipulatives that are utilized in my classroom initially. I plan to use some of the premade activities from the books *Exploring Algebra and Pre-algebra with Manipulatives* (Balka, 1995), *Hands-On Algebra! Ready to Use Games & Activities for Grades 7-12* (Thompson, 1998), *Navigating through Algebra in Grades 3-5* (Cuevas & Yeatts, 2001), and *Navigating through Algebra in Grades 6-8* (Friel et al., 2001). I also plan to create some of my own activities. With the LCD projector in my classroom and access to laptop computers, I will easily be able to incorporate the use of virtual manipulatives into my curriculum drawing from online collections. I plan to continue to research virtual manipulatives as they are created online. I am currently pursuing the purchase and implementation of Algeblocks and a SMART board for use in my classroom.

Getting Started with Manipulatives

If you are an instructor that does not have a background or experience with the use of manipulatives in the classroom, you are not alone. NCTM (2000) states that preservice preparation lays the foundation for mathematics instruction, but it is only a small part of what teachers need to know and understand throughout their career. They recommend that teachers develop their own professional knowledge using research as well as using their own experiences. Teachers continually need professional development in order to deliver high-quality mathematics instruction. They must have frequent and ample opportunity and resources in order for their knowledge to grow.

I felt I needed to change something in my teaching in order to increase student understanding and achievement. Colleagues that have started to incorporate manipulatives into their classroom have done it for similar reasons. Similarly, NCTM (2000) tells us that it is important for instructors to be able to describe and explain why they are working toward a specific goal.

I have done a significant amount of research on the use of manipulatives, but in my opinion, I have only just begun. I know that I will need to make a continued effort to keep learning as things continue to change and develop. The manipulatives that I use in my classroom must be subject to ongoing reflection and modification.

I have been using my own research to improve my teaching, but my participation in professional development has been minimal. NCTM (2000) states that the present day practice of offering occasional workshops and in-service days is not enough. Clearly it has not been enough for me. I need to make an effort to partake in as many professional development opportunities as I can for myself as well as my students.

How do I Plan to Use this Research?

Initially, I will incorporate the use of manipulatives throughout my Algebra 1A curriculum. I plan to research and implement the use of manipulatives into my other classes, especially in Pre-algebra and Algebra 1B. I aim to continue to compile data from all mathematics students, and with the help of the administration and the mathematics department, look for ways to improve my teaching and student achievement.

During the coming school year, I would like to share my findings with the other Algebra 1A instructors. I will share the activities that I use in my Algebra 1A class with the other instructors that are concurrently teaching Algebra 1A. Beyond my own drive, there is currently a push in our high school to help the students at this level to become more successful and to gain a deeper understanding of the material. On a trial basis during the 2010-2011 school year, two Algebra 1A classes are going to be split in one class period, thus, students will have Algebra 1A for forty-five minutes all year long. I hope to work closely with the instructors of these two classes as these students were placed here due to low standardized test scores. I hope that my findings will help the instructors as well as the students to achieve the greatest levels of success possible. As recommended by NCTM (2000), I hope to collaborate with my coworkers and partake in lesson studies geared toward the Algebra 1A curriculum.

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This virtual manipulative incorporate the use of Algebra tiles in solving one and two step linear equations with variables on one or both sides. The user can input an equation or request a premade equation.

Applets. (2010). *For Java Developers*. Retrieved August 12, 2010, from http://java.sun.com/applets/

This site defines an applet.

Burns, M. (2004). 10 big math ideas. *Instructor (New York, N.Y.: 1999), 113*(7), 16-19, 60. Retrieved August 2, 2008, from Wilson Web database.

The author gives ten reasons—with supporting information—that can be used to help students with learning. Some of the reasons will help support the research that the use of manipulatives increases learning, while one of the reasons is just that: “Support learning with Manipulatives.” The author also includes a third grade lesson plan which is irrelevant and will be of no use.

Balka, D. S. (1983). *Mathematics manipulaitves in a pre-vocational program: Teacher in-service and classroom research.* City, AA: Publisher. (ERIC Document Reproduction Service No. ED237739)

This is a small study that was done on mildly mentally handicapped high school students. Manipulatives were used in the classroom and shown to have positive results. This was used with a basic math curriculum though and not algebra.

Balka, D. S. (1995). *Exploring algebra and pre-algebra with manipulatives*. Rowley, MA: Didax.

This book gives hard copies of games and manipulatives that can be duplicated for use in the algebra and pre-algebra classroom. The instructor will need to create or supply their own manipulatives required, but instructions are given instruction on how to make your own if necessary.

Center for Technology in Learning. (2007). *SimCalc: Research*. Retrieved June 28, 2010, from http://math.sri.com/research/ index.html

This page gave a list of all the groups involved in the SimCalc research projects.

Chappell, M. F., & Strutchens, M. E. (2001). Creating connection: Promoting algebraic thinking with concrete models. *Mathematics Teaching in the Middle School*, *7*(1), 20-25. Retrieved August 3, 2008, from Wilson Web database.

This journal talks about the use of manipulatives. It discusses several lessons that use algebra tiles and how they relate to a specific abstract concept. Gives some examples of student misconceptions, but seems rather vague.

Clements, D. H. (1999). "Concrete" manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60. Retrieved June, 27, 2010, from http://gse.buffalo.edu/org/buildingblocks/Newsletters/Concrete_Yelland.htm

This article discusses the thought process behind using manipulatives. It discusses a specific virtual manipulative, *Squares*, and all of its benefits. It also discusses numerous advantages computer based manipulatives have over physical manipulatives.

Craighead, D. (2004). High school math. *Technology & Learning*, *24*(7), 26. Retrieved August 02, 2008, from GALE database.

The author lists multiple online sources where manipulatives can be found and a short description of the site. This will be a great place to start building a "tool chest" of manipulatives as the author refers to it.

Cuevas, G. J., & Yeatts, K. L. (2001). *Navigating through algebra in grades 3-5*. Reston, VA: National Council of Teachers of Mathematics.

This book is part of the Navigation Series from NCTM. It contains activities in patterns, variables and equations, and functions. It includes instructions for teachers as well as any handouts that are necessary for students.

Curriculum, Technology, & Education Reform. (2008). *Manipulatives in Mathematics Instruction.* Retrieved August 6, 2008, from http://wik.ed.uiuc.edu/index.php/ Manipulatives_in_Mathematics_Instruction

In reference to mathematical manipulatives, this article includes a definition, application in the classroom, two ways effectiveness is measured, which was very insightful, and what critics say about their use. It also contains multiple personal testimonies in reference to the positive and negative aspects of using manipulatives. This article has a lot of statements that will support this research although it is not backed by any specific research.

Dienes, Z. P. (1967). *Building up mathematics*. London: Hutchinson Educational.

Dienes discusses the position of mathematics education in the 1950's. Even though it is a half century later, many of observations on the problems can be related to current problems of mathematics education. It is interesting how far math education has come in that time period, but yet how far we still need to go. He also presents his theory of mathematics-learning. I found both of these chapters to be useful. The rest of the book discusses examples to be used in the classroom. I don't think there will be much that I can use directly in my classroom at this point in time from these chapters.

Dienes, Z. (n.d.). Zoltan Dienes' six-stage theory of learning mathematics. *Zoltan Dienes' Web Site — Biography, Math Games, Poetry and more...* Retrieved June 25, 2010, from http://www.zoltandienes.com/?page_id=226

Dienes web site lists his six-stage theory of learning. It is a modern extension of his learning theory that he discussed in his book.

Deubel, P. (2010). *Math Manipulatives.* Retrieved from June 27, 2010, http://www.ct4me.net/math_manipulatives.ht

Valuable web site. It has great summaries as well as a very well organized summary of links to math manipulative sites.

Eisenberg, M., & DiBiase, J. (1996). Mathematical manipulatives as designed artifacts: The cognitive, affective, and technological dimensions. *International Conference on Learning Sciences,* 44-51. Retrieved August 3, 2008, from the ACM Digital Library database.

This article provides information on the types of learning the manipulatives are intended to promote, the recent move toward technological manipulatives and the social aspect of the use of manipulatives. It also lists some possibilities for primary research. It reported on scheme paint and HyperGami neither of which have much application in algebra, but the author makes a good point about using manipulative creation tools over regular manipulatives when applicable. This could provide to be a very useful resource.

Eremin, E. (2007). About the experience of developing interactive dynamic illustraions for educational e-materials. *Information Technologies and Knowledge*, *1*, 53-57. Retrieved June 22, 2010, from http://hdl.handle.net/10525/104

This journal discusses the development of illustrative dynamic material for both specific and universal instruction.

Friel, S. N., Rachlin, S., & Doyle, D. (2001). *Navigating through algebra in grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

This book is part of the Navigation Series from NCTM. It contains activities in understanding patterns, relations, and functions; analyzing change in various contexts; exploring linear relationships; and using algebraic symbols. It includes instructions for teachers as well as any handouts that are necessary for students.

Garrity, C. (1998). *Does the use of hands-on learning, with manipulatives, improve the test scores of secondary education geometry students?* Unpublished master's thesis, St. Xavier University. Chicago, IL. Retrieved June 26, 2010, from http://www.eric.ed.gov/ERICWebPortal/search/recordDetails.jsp?searchtype=keyword&pageSize=10&ERICExtSearch_SearchValue_0=ED422179&eric_displayStartCount=1&ERICExtSearch_SearchType_0=no&_pageLabel=RecordDetails&objectId=0900019b800b67db&accno=ED422179&_nfls=false

This is the thesis on the action research done with the use of manipulatives in two secondary geometry courses. Her data did not show significant gains in student grades, but her results did indicated an increase in positive student attitudes towards mathematics.

Haas, M. (2005). Teaching methods for secondary algebra: A meta-analysis of findings. *Nassp Bulletin, 89*(642), 24-46. Retrieved August 2, 2008, from SAGE Education database.

This article is a meta-analysis of thirty five independent experimental studies. It includes details from the different experiments as well as the conclusions drawn from them. It concludes that six teaching methods (cooperative learning; communication and study skills; technology-aided instruction; problem-based learning; manipulatives, models, and multiple representations; and direct instruction) increase student understanding. This is a great piece of secondary research on algebra instruction which will hopefully lead to some reliable primary research that supports the use of manipulatives.

Hall, B. C. (1994). *Developing concepts using algebra tiles*. Englewood Cliffs, NJ: Prentice Hall.

This book contains premade lessons for Algebra tiles for adding, subtracting, multiplying and dividing integers; the distributive property; representing algebraic expressions; solving linear equations; adding, subtracting, and multiplying polynomials; factoring; and solving quadratic equations.

Hinzman, K. P. (1997). *Use of manipulaitves in mathematics at the middle school level and their effects on students' grades and attitudes* Unpublished master's thesis, Salem-Teikyo University, Salem, WV. Retrieved June 26, 2010, from http://www.eric.ed.gov/ERICWebPortal/search/recordDetails.jsp?searchtype=keyword&pageSize=10&ERICExtSearch_SearchValue_0=ED411150&eric_displayStartCount=1&ERICExtSearch_SearchType_0=no&_pageLabel=RecordDetails&objectId=0900019b800ae8ec&accno=ED411150&_nfls=false

This is the thesis on the action research done with the use of manipulatives in two eighth grade pre-algebra courses. It is hard to read at times—especially the data. Her data did not show significant gains in student grades, but her surveys and observations did show that the use of manipulatives had a positive effect on the feelings and attitude towards math.

Illuminations: Activities. (n.d.). *Illuminations: Welcome to Illuminations*. Retrieved June 27, 2010, from http://illuminations.nctm.org/ActivitySearch.aspx

This web site contains over one hundred online activities broken down by grade to be used in the classroom.

Jones, S. (1986). The role of manipulatives in introducing and developing mathematical concepts in elementary and middle grades. *Resource Room: Free-Spirited Structured, Multisensory Learning*. Retrieved June 23, 2010, from http://www.resourceroom.net/math/Jones_mathmanip.asp

This paper gives an analysis of research about using manipulatives to teach math. The author gives reason why manipulatives may not help students. She gives some insights into research that I have not come across. Title refers to elementary and middle school, but she refers to secondary school in her paper.

Larson, L. (2009, September). *Minnesota's high school graduation requirements*. St. Paul: Minnesota, House of Representatives. Retrieved June 22, 2010, from http://www.house.leg.state.mn.us/hrd/pubs/ss/sshsgrad.pdf

This journal is a summary of all the graduation requirements for students in the state of Minnesota. It includes credit requirements, including specific required courses, as well as the required state tests.

Larson, R. (2001). *Algebra 1*. Evanston, IL: McDougal Littell

This is an Algebra tiles investigations book has ten investigations that includes teacher notes, a classroom activity and practice questions.

Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). *Algebra 1*. Evanston, IL: McDougal Littell.

This is the text book that is used at Bemidji High School for the Algebra 1A and 1B courses.

Lindroth, L. (2005). How to…find online math manipulatives. *Teaching preK-8, 35*(4), 24-26. Retrieved August 2, 2008, from Wilson Web database.

The author lists reasons for the use of virtual manipulatives and lists multiple online sources where manipulatives can be found and a short description of the site. The involvement of parents is also discussed. This site will be a great resource when trying to find manipulatives to be used in the classroom.

Maccini, P., & Ruhl, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, *23*(4), 465-489. Retrieved August 3, 2008, from ACM Digital Library database.

This journal discusses a research project that was done with using manipulatives with LD students. It was a very small study but it had positive results.

Marshall, P. (2007). Do tangible interfaces enhance learning? In *Proceedings of the 1st international conference on Tangible and embedded interaction, February 15-17, 2007, Baton Rouge, Louisiana* (pp. 163-170). New York: ACM Press. Retrieved August 3, 2008, from ACM Digital Library.

This paper discusses the recent creation and developments of digital manipulatives and their benefits in learning. It is not specific to any one curriculum. It states that not much research has been done, and suggests that more research is needed to guide developments of this new branch of manipulatives.

McClung, L. W. (1998). *A study on the use of manipulatives and their effect on student achievement in a high school algebra 1 class.* Unpublished master's thesis, Salem-Teikyo University, Salem, WV. (ERIC Document Reproduction Service No. ED 425077)

McClung conducted a study on the use of Algeblocks in his Algebra 1 course. The control group which was taught using lecture and practice outperformed the group using the manipulative. The author admitted that this result may have been due to his inexperience in using the manipulative.

Minnesota Department of Education. (2008). *BST*. Retrieved August 7, 2008, from http://cfl.state.mn.us/mde/Accountability_Programs/Assessment_and_Testing/Assessments/BST/index.html

This site defines the Basic Skills Tests in Minnesota.

Minnesota Department of Education. (2008). *MCA-II*. Retrieved August 7, 2008, from http://education.state.mn.us/mde/Accountability_Programs/Assessment_and_Testing/Assessments/MCA_II/index.html

This site defines the Minnesota Comprehensive Assessments—Series II tests.

Minnesota Department of Education. (2010). *Adequate Yearly Progress*. Retrieved June 22, 2010, from http://education.state.mn.us/mde/Data/Data_Downloads/Accountability_Data/NCLB_AYP/index.html

This site defines the term Adequate Yearly Progress.

NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

This book updates the NCTM's previous Standards and breaks them down by grade bands–pre-K–2, 3–5, 6–8, and 9–12. It contains some good information regarding professional development. The accompanying CD offers a searchable hypertext edition of Principles and Standards, which includes interactive math tools that support the Standards. The CD also contains a version of the Illuminations Web site, which includes lessons, interactive tools, and reviewed Web sites that support Principles and Standards and the three previous Standards documents.

Project Interactivate. (n.d.). *Shodor education foundation: A national resource for computational science education*. Retrieved July 12, 2010, from http://www.shodor.org/interactivate/

Project Interactivate's goals included the "creation, collection, evaluation, and dissemination of interactive Java-based courseware for exploration in science and mathematics. The site includes 70 NCTM based lesson plans, a dictionary of math terms used throughout the site, and over 100 Java-based interactive activities. These activities are broken down by content area and grade level.

Raymond, A. M., & Leinenbach, M. (2000). Collaborative action research on the learning and teaching of algebra: a story of one mathematics teacher's development. *Educational Studies in Mathematics*, *41*(3), 283-307. Retrieved August 3, 2008, from Wilson Web database.

This paper is the summary of collaborative action research that was done between an eighth grade algebra teacher and a university researcher. A sub study of the action research investigated the teaching and learning of algebra using manipulatives compared to the traditional method using the textbook. The findings in the classroom had positive results, but the long term affects were inconclusive.

Reed, S. K. (2005). From research to practice and back: The animation tutor project. *Educational Psychology*, *17*(1), 55-82. Retrieved August 3, 2008, from Wilson Web database.

This journal explains Animation Tutor and its history. Research was used to create modules, virtual manipulatives, that would aid in the instruction of courses such as intermediate algebra. It explains each module and the research that was done to improve each one. They are available free online for download.

Resnick, M., Martin, F., Berg, R., Borovoy, R., Colella, V., Kramer, K., et al. (1998). Digital manipulatives: New toys to think with. *Conference on Human Factors in Computing Systems,* 281-287. Retrieved August 3, 2008, from ACM Digital Library database.

This paper discusses "digital" manipulatives. After giving a brief synopsis of the origin of hand-held manipulatives it reports on the digital manipulatives that the researchers (authors) created. This was written in 1998 and as technology has changed drastically in the last ten years, it would be interesting to see if they have any new research. None of the manipulatives they developed are applicable in algebra.

Roschelle, J., Schechtman, N., Tatar, D., Hegedus, S., Hopkins, B., Empson, S., et al. (2010). Integration of technology, curriculum, and professional development for advancing middle school mathematics: Three large-scale studies. *American Educational Research Journal*. Retrieved June 28, 2010, from http://ctl.sri.com/publications/downloads/SimCalc AERJ_January2010nikki.pdf

This paper discusses the experiments that were done with SimCalc, a virtual manipulative. All results showed that the use of this virtual manipulative cause increased student academic achievement. It is geared toward the algebra strand and is available online. May be a good manipulative to incorporating in my classroom.

Stewart, M. (2003). From tangerines to algorithms. *Instructor (1990)*, *112*(7), 20-23. Retrieved August 02, 2008, from GALE database.

The author has a collection of personal testimonies from math instructors and other experts attesting to the positive results of the use of manipulatives in math instruction. The point is made that that as difficulty of mathematical concepts increases, the use of manipulatives in the classroom decreases. The article essentially only contains opinions as there is no research other that personal experience to support the findings, but it also includes some good ideas for manipulatives to use in the classroom as well as good implementation suggestions. Some of the opinions that were presented in the article as direct quotes could be used so support the research that finds an increase in student understanding due to the use of manipulatives. More research on the experts that were presented could follow from this article.

Strom, J. (2009). *Manipulatives in mathematics instruction.* Unpublished master's thesis, Bemidji State University, Bemidji, MN. Retrieved July 2, 2010, from http://faculty.bemidjistate.edu/grichgels/MastersPapers/Strom,_Jessica.pdf

This paper looks at the research surrounding manipulatives as well as the author's experiences with manipulatives. Her account of use with Algeblocks is of interest to me.

Thompson, F. M. (1998). *Hands-on algebra!: Ready-to-use games & activities for grades 7-12*. West Nyack, NY: Center for Applied Research in Education.

Each objective in the book is presented in a series of three lessons or activities. Activity 1 involves physical models in the development of the concept. Activity 2 uses pictorial or graphic models to help students bridge from the concrete to the abstract. Students then draw conclusions and make generalizations from these activities. Activity 3 allows students to further explore these generalizations. The manipulatives involved, the instructor's role, any student worksheets, and the approximate time that the activity should take are all included in the book. The manipulatives involved are all things that can be created out of inexpensive items. There are templates included for out of the ordinary things that are used.

Trotter, A. (2007). R & d project on algebra software seen to show promise. *Education Week, 27*(5), 10. Retrieved August 2, 2008, from Wilson Web database.

This article looks at virtual manipulatives, specifically the mathematical computer software SimCalc, as well as the use of graphing calculators in conjunction with TI-Navigator. Studies continue to be conducted in Texas and Massachusetts where both showed promising results within all levels of algebra learners. The software is available free online and is intended to be used in Algebra I & II. The details of the Texas study were reported, but I would like to see if I could find the primary research on both studies as they are still ongoing. I could use the positive results of the studies to reinforce my findings as well as look into possibly using this resource in my classroom. I cannot rely on it entirely as it is not the primary research.

U.S. Department of Education. (2008). *No Child Left Behind Act of 2001*. Retrieved August 7, 2008, from http://answers.ed.gov/cgi-bin/education.cfg/php/enduser/std_adp.php?p_faqid=4

This site defines the No Child Left Behind Act of 2001.

Utah State University. (2010). *National Library of Virtual Manipulatives*. Retrieved June 23, 2010, from http://nlvm.usu.edu/

The National Library of Virtual Manipulatives is a National Science Foundation supported project conducted by Utah State University that began in 1999 in which a library of virtual manipulatives encompassing K-12 mathematics instruction was created. It is broken down by both grade level and contents area including Number & Operations, Algebra, Geometry, Measurement, and Data Analysis & Probability.

Waite-Stupiansky, S., & Stupiansky, N. G. (1998). Hands-on, minds-on math. *Instructor (1990)*, *108*(3), 85. Retrieved August 02, 2008, from GALE database.

This site defines the No Child Left Behind Act of 2001.

Zuckerman, O., Arida, S., & Resnick, M. (2005). Extending tangible interfaces for education: Digital Montessori-inspired manipulatives. In *CHI 2005, connect: conference proceedings: Conference on Human Factors in Computing Systems, April 2-7, Portland, Orgeon* (pp. 859-868). New York: ACM Press. Retrieved August 3, 2008, from ACM Digital Library database.

This paper discusses concrete manipulatives and gives them their own classifications. It discusses the differences of the two types and gives specific examples. They conducted studies with these different manipulatives and reported their findings.

Mathematics Teacher Mathematical Knowledge Alternative Conception Mathematics Classroom Mathematical Activity.Â Using history of mathematics in junior secondary school classroom: a curriculum perspective (in Chinese), MPhil Thesis, Hong Kong: Chinese University of Hong KongGoogle Scholar. Lombardo Radice, L. 1981. Ä½infinito, Roma: Editori Riuniti.Google Scholar.Â â€˜The ABCD of using history of mathematics in the (undergraduate) classroomâ€™, Bull. Hong Kong Math. Soc. From algebra and geometry comes calculus. This is the most important tool that mathematicians have to describe change, for example, if you calculate the speed of a moving car or analyse the way the population of a city changes over time. The most significant area of calculus is function, which is concerned with the relationship between argument and result.Â How do you use mathematics outside the classroom? How is mathematics involved in cooking? Ð' F Listening N))). Listen to these people talking about mathematics in everyday life.Â â€¢ how they might be of use in the real world. â€¢ Read the texts again and use these. notes to help you. N. The use of tools with capabilities for algebra, geometry and calculus would require serious consideration of the place of techniques and concepts in DP mathematics and it may be useful to consider parallel by-hand/by-technology DP courses so that teachers may come on board when they are confident3.Â (2012), From text to â€˜livedâ€™ resources: Mathematics curriculum materials and teacher development.Â 1. Teacher Issues and Technology Use a) Some Theoretical Considerations b)Â a) Some Theoretical Considerations There have been a number of theoretical approaches applied to issues surrounding technology use in the secondary mathematics classroom and the last decade has seen efforts aimed at using these frameworks to provide different, complementary views of the same subject.