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# **Quantum Transport and Dissipation**

## Preface

In recent years, systems intermediate between macroscopic and microscopic scales, where quantum coherence becomes important, have developed into a main area of research in modern condensed-matter physics. The notion of “mesoscopic systems” has been coined to characterize this field. One of its fascinating aspects is that it connects quite different subjects ranging from quantum transport theory to quantum chaos. Here, technical applications, for instance single-electron transistors, are closely tied to fundamental questions in quantum and statistical mechanics. As a result of the rapid growth of the literature, it has become very difficult for young scientists to obtain an overview of the foundations of this interdisciplinary field.

The aim of this book is to provide an introduction into some of the theoretical aspects. At the same time, we intend to guide towards the forefront of ongoing research in an immensely vivid subject. This goal is almost impossible to achieve. We can only hope that the compromise attempted here will prove successful.

In order to cover the wide range of topics, the book has been written by several authors. Each of the six chapters is aimed at a reasonably self-contained introduction into the principal issues of a particular subfield, which we believe to be representative. The first chapter deals with the basic notions in quantum transport theory, including weak localization, universal conductance fluctuations and the Landauer-Büttiker scattering formalism. It also provides some background for diagrammatic techniques in weakly disordered systems. The second chapter introduces the theory of the quantum Hall effect, both integer and fractional. The concept of localization as well as Laughlin’s theory of the effect of interactions in the fractional case are treated here on an elementary level. Quantization of conductance in quantum wires, including the effects of impurity scattering and interactions, is outlined here. The subject of the following chapter are single-electron tunneling and Coulomb-blockade phenomena for metallic and superconducting junction devices. Beyond the standard rate equation, it contains a discussion of the effect of the electromagnetic environment and of nonperturbative effects.

In chapter four, concepts for the description of dissipation in quantum systems, in particular the role of the coupling to a heat bath, are explained. This chapter also includes the Feynman path-integral formulation via influence functionals and applies this formalism to dissipative tunneling. The fifth chapter is devoted to the theory of strongly driven quantum systems. On basis of the general Floquet theory, archetypal problems like driven tunneling—together with its dynamical destruction—in a driven double well and the control of a quantum dynamics via a sequence of coherent laser pulses are discussed there. Quantum chaos and its relation to transport phenomena are treated in the final chapter. The main topics are semiclassical methods, the interplay of chaos with tunneling and localization, and quantum chaos in scattering and dissipative systems.

The present volume introduces many of the basic ideas and techniques in this field which are necessary to master the more specialized original literature. As such it is mainly addressed to those who intend to start research in this very active area. It

should also be useful for physicists in other fields who would like to learn about this subject or to experimentalists for a better understanding of the crucial theoretical concepts. It is obvious that many more interesting topics like persistent currents and the physics of semiconductor quantum dots could not be included or could only be touched very briefly. Nevertheless, we are confident that the present volume reflects at least the cornerstones of the physics of mesoscopic systems and conveys the excitement in this area of research.

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